

THORN Planning Memo – tpp / ntp Case

1. Core Observation

THORN proves $A \blacksquare B$ by assuming A and deriving B using sequent reasoning. Definitions introducing disjunctions and existentials cause search explosion.

2. Problem

Goal: $tpp(x,y) \ \& \ ntp(y,z) \Rightarrow ntp(x,z)$

$ntp(x,z) = pp(x,z) \ \& \ \neg \exists u (ec(u,x) \ \& \ ec(u,z))$

3. Strategy

Use definition-driven recursive decomposition instead of trivial lemmas plus one large theorem.

4. Reduction

Core lemma: $tpp(x,y), ntp(y,z), ec(u,x) \blacksquare \neg ec(u,z)$

5. Key Insight

Difficulty arises from missing bridge lemmas relating containment (c), overlap (o), and tpp/ntp.

6. Failure Mode

Trivial lemmas succeed quickly; final lemma explodes due to existential witnesses and branching.

7. Distinction

THORN derives sequents $\Gamma \blacksquare A$, not contradiction-based proofs.

8. Hard Cases

tpp with ntp, inverse relations, and ec witnesses.

9. Open Problem

Main unresolved lemma: $tpp \ x \ y \ \& \ ntp \ y \ z \ \& \ ec \ u \ x \Rightarrow \neg \ ec \ u \ z$

10. Takeaway

Reduce to one hard lemma; avoid large monolithic goals.